

QCD instantons in high energy diffractive scattering: Instanton model of Pomeron*

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The role of the QCD vacuum effects in high energy diffractive quark-quark and quark-antiquark scattering is studied with the Instanton Liquid Model. Special attention is given to the problem of formation of the soft Pomeron. We show that in the leading approximation in instanton density the C -odd instanton contribution to the diffractive amplitude is suppressed by $1/s$ compared to the C -even one.

Strong interaction processes in the Regge regime: $s \gg -t$, *i.e.*, collisions with large total center-of-mass energy and small momentum transfer, yield the main contribution to the hadronic cross sections at high energy. These processes are described successfully within the Regge phenomenology, with the Pomeron exchange being dominant in this regime [1]. The Pomeron is treated as an effective exchange in the t channel with vacuum quantum numbers and positive charge parity $C = +1$. Within perturbative QCD, the pioneer calculations of the Pomeron properties were performed in Refs. [2], which lead to the supercritical “hard” Pomeron violating the Froissart bound. Further, it was shown that the NLO corrections can change significantly this leading logarithmic approximation result [3]. From the other point of view, it is natural to expect that the QCD dynamics at large distances and the nontrivial structure of the QCD vacuum are relevant for such processes with small momentum transfer [4]. A fruitful approach is to

try to reformulate the complicated QCD dynamics in terms of some effective theory which would be easier to solve in a given regime. The convenient effective degrees of freedom at high energy are the Wilson path-ordered exponentials evaluated along the straight-line trajectories of colliding particles [5–8].

In this work, we consider the high energy diffractive quark-quark scattering using the Instanton Liquid Model (ILM) of the QCD ground state [9–11] in order to take into account nonperturbative effects in formation of the soft Pomeron. A similar situation was first analysed in Refs. [12, 13] from a somewhat different point of view. The quark-quark scattering amplitude is expressed in terms of the vacuum average of the gauge invariant path-ordered Wilson exponential [5,6,14]

$$T_{mn}^{kl}(s, t) = -2is \int d^2b_{\perp} e^{ib_{\perp}q} W_{mn}^{kl}(\chi, b_{\perp}^2), \quad (1)$$

where the Wilson integral W_{mn}^{kl} reads

$$W_{mn}^{kl}(\chi, b_{\perp}^2) = \left\langle \mathcal{P} e^{ig \int_{C_{qq}} dx_{\mu} \hat{A}_{\mu}(x)} \right\rangle \bigg|_{mn}^{kl}. \quad (2)$$

In Eq. (2), the corresponding integration path goes along the closed contour C_{qq} : two infinite lines separated by the transverse distance b_{\perp} and

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having relative scattering angle χ . We parameterize the integration path as $C_\chi = \{z_\mu(\lambda); \lambda = [-\infty, \infty]\}$ where $z_\mu(\lambda) = v_1 \lambda$, $-\infty < \lambda < \infty$, and $z_\mu(\lambda) = v_2 \lambda + \mathbf{b}$, $-\infty < \lambda < \infty$. with $(v_1 v_2) = \cosh \chi$ and $\mathbf{b} = (0_\parallel, b_\perp)$ being the impact parameter in the transversal plane. To calculate the amplitude (1) in the instanton background, we use the explicit expression for the instanton field

$$\hat{A}_\mu(x; \rho) = \frac{1}{g_s} \mathbf{R}^{ab} \sigma^a \eta^{\pm b}_{\mu\nu} (x - z_0)_\nu \varphi(x - z_0; \rho). \quad (3)$$

The averaging $\langle \dots \rangle_0$ over the nonperturbative vacuum consists in integration over the coordinate of the instanton center z_0 , the color orientation \mathbf{R}^{ab} and the instanton size ρ : $dI = d\mathbf{R} d^4 z_0 dn_\rho$, where the instanton size distribution dn_ρ is chosen according to ILM as [9] $dn_\rho = n_c \delta(\rho - \rho_c) d\rho$, $n_c \approx 1 fm^{-4}$, $\rho_c \approx 1/3 fm$. Evaluating the path-ordered exponential Eq. (2) and averaging over all possible embeddings of $SU_c(2)$ into $SU_c(3)$ by using the relations from Ref. [15] we get (for further technical details, see Ref. [16])

$$W_{mn}^{kl}(\gamma, \mathbf{b}^2) = n_c \left\{ \frac{4}{9} \delta_{kl} \delta_{mn} w_c(\gamma, \mathbf{b}^2) + \frac{1}{8} \lambda_{kl}^a \lambda_{mn}^a \left[\frac{1}{3} w_c(\gamma, \mathbf{b}^2) + w_s(\gamma, \mathbf{b}^2) \right] \right\}, \quad (4)$$

$$w_c(\gamma, \mathbf{b}^2) = \int d^4 z_0 (\cos \alpha_1 - 1) (\cos \alpha_2 - 1), \quad (5)$$

$$w_s(\gamma, \mathbf{b}^2) = - \int d^4 z_0 (\hat{n}_1^a \hat{n}_2^a) \sin \alpha_1 \sin \alpha_2, \quad (6)$$

where the color correlation factor is

$$\hat{n}_1^a \hat{n}_2^a = \frac{(v_1 v_2)(z_0, z_0 - \mathbf{b}) - (v_1 z_0)(v_2 z_0)}{s_1 s_2}. \quad (7)$$

The phases are defined as

$$\alpha_1 = s_1 \cdot \int_{-\infty}^{\infty} d\lambda \varphi[(z_0 + v_1 \lambda)^2; \rho], \quad (8)$$

$$\alpha_2 = s_2 \cdot \int_{-\infty}^{\infty} d\lambda \varphi[(z_0 - v_2 \lambda - \mathbf{b})^2; \rho]. \quad (9)$$

with $s_1^2 = z_0^2 - (v_1 z_0)$; $s_2^2 = (z_0 - \mathbf{b})^2 - (v_2 z_0)$. Here γ is the scattering angle in Euclidean space, whereas in the final expressions one must make a

transition back to Minkowski space-time (see below). By means of the proper change of variables, the energy dependence is trivially factorized

$$w_c(\gamma, b_\perp^2) \rightarrow \frac{1}{\sin \gamma} w_c(\pi/2, b_\perp^2), \quad (10)$$

$$w_s(\gamma, b_\perp^2) \rightarrow \cot \gamma w_s(\pi/2, b_\perp^2). \quad (11)$$

The differential cross section of the quark-quark scattering is expressed through the amplitude (4) as

$$\frac{d\sigma_{qq}}{dt} \approx \frac{1}{9} \frac{1}{s^2} \sum_{kl} \sum_{mn} |T_{mn}^{kl}(s, t)|^2. \quad (12)$$

Making analytical continuation to Minkowski space [17,18]: $\gamma \rightarrow -i\chi$, one finds

$$\begin{aligned} \frac{d\sigma_{qq}(t)}{dt} = & \frac{2}{9} n_c^2 [\coth^2 \chi F_s^2(t) + \\ & + \frac{2 \coth \chi}{3 \sinh \chi} F_c(t) F_s(t) + \frac{11}{3} \frac{1}{\sinh^2 \chi} F_c^2(t)] , \end{aligned} \quad (13)$$

where

$$F_{s,c}(t) = \int d^2 \mathbf{b} e^{i \mathbf{b} \mathbf{q}} w_{s,c}(\pi/2, \mathbf{b}^2). \quad (14)$$

In the asymptotic limit ($\sinh \chi \sim s$, $\coth \chi \rightarrow 1$) the result (13) coincides with the result of Ref. [12]: $\frac{d\sigma}{dt} \approx \frac{2}{9} n_c^2 F_s^2(t)$. In the weak field limit we reproduce the one-loop single instanton results (see Ref. [16]).

For the quark-antiquark scattering one can treat an antiquark with velocity v_2 as a quark moving backward in time with velocity $-v_2$. As a result, the scalar product of velocities changes its sign ($v_1^q v_2^{\bar{q}} = -(v_1^q v_2^q)$) and the scattering angles are related as $\chi_{qq} \rightarrow i\pi - \chi_{q\bar{q}}$. Then one gets

$$\begin{aligned} \frac{d\sigma_{q\bar{q}}(t)}{dt} = & \frac{2}{9} n_c^2 [\coth^2 \chi F_s^2(t) - \\ & - \frac{2 \coth \chi}{3 \sinh \chi} F_c(t) F_s(t) + \frac{11}{3} \frac{1}{\sinh^2 \chi} F_c^2(t)] . \end{aligned} \quad (15)$$

The second terms in Eqs. (13) and (15) correspond to the contribution of the C -odd amplitude.

The spin averaged total quark-quark cross section in the instanton–antiinstanton approximation reads

$$\sigma_{qq} \approx \frac{2}{9} n_c^2 \int_0^\infty dq^2 \left[F_s^2(q^2) + \frac{4}{3} \frac{m^2}{s} F_c(q^2) F_s(q^2) \right] \quad (16)$$

which is constant in the high energy limit. It is finite if the constrained instanton solution is used [19]. In Eq. (16), the only term corresponding to the $C = +1$ exchange, Pomeron, survives in the asymptotics, while the $C = -1$ contribution (second term in Eq. (16)) is suppressed by the small factor $\sim m^2/s$. The leading $C = -1$ part of the scattering amplitude, odderon, will arise at higher orders in instanton density, which corresponds to diagrams like three nonperturbative gluon exchange. The growing part of the total cross section $\Delta\sigma_{qq} \sim (n_c \rho_c^4)^2 \Delta(t) \ln s$ can arise only if *inelastic* quark-quark scattering in the instanton-antiinstanton background is taken into account [12].

It is important to note that the original Wilson exponential, Eq. (2), has essentially Minkowskian light-cone geometry whereas the instanton calculations of Wilson loop are performed in the Euclidean QCD. The mapping from the Minkowski space to the Euclidean one is possible since the dependence of the Wilson integral on the total energy s and transverse momentum squared t is factorized in Eqs. (10), (11). At high energy, the amplitude is s –independent both in the perturbative and nonperturbative cases. At the same time, the t –dependence of the amplitude is naturally expressed through the nonperturbative instanton field.

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